siderably improved by perhaps an order of magnitude. Even then, the theoretical uncertainties are much too great at present. It would be necessary to develop some reliable method of generating an isotopic vector current which accounts correctly for the magnetic moment. The method used thus far, that of Sachs', gives a result for this in the right direction but an order of magnitude too small. This is the most unreliable facet of any calculation of these nuclear form factors. With the provision that Sach's method of including this exchange current is reasonable, some conclusions may be drawn about the nuclear wave functions. From the curves above, it is possible to fit well the shape of the cross sections for He^3 and H^3 separately with a mixture of S and D states. These fits, however, are not good if the same parameters are chosen for each nucleus, the falloff distance in H^3 being considerably smaller than in He^3. It is doubtful whether the Coulomb repulsion between protons giving a Coulomb energy on 0.764 MeV could account for this; using the wave function from curve J the Coulomb energy for point nucleons is 0.743 MeV. The value for extended nucleons is expected to be 0.669 to 0.600 MeV or 10–20% less on the basis of calculations done by Ohmura and Ohmura.[15] Thus, while a purely symmetric S state is ruled out by the differences in F_{chg} and F_{mag} for He^3, neither can a mixture of S and D states give these form factors correctly for both nuclei, with the same parameters. The differences in finite sizes effects between proton and neutron accounts partially for this difference, but only for about half of it.

Finally, Table II summarizes the best values of the parameters from these curves, and contrasts them with the corresponding quantities found by Schiff and with those found in other types of experiments.

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Low-Lying Collective States of Sm^{152} and Sm^{154}†

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The excited states up to 2 MeV in Sm^{152} and Sm^{154} have been studied by magnetic analysis of the inelastic protons from thin targets bombarded by a 12-MeV proton beam from the Florida State University tandem Van de Graaff accelerator. The results are compared with previous studies and with the predictions of collective nuclear models. The ground-state band levels up to spin 6 are excited in these experiments. The 2+, 3+, and 4+ states in the gamma vibrational band, the 0+ and 2+ states in the beta vibrational band, and the 1−, 3−, and 5− states in the octupole band have been observed in Sm^{152}. In Sm^{154} the gamma band head is observed at 1443 keV and the octupole band head is observed at 927 keV. Additional levels in these bands and the beta band are suggested. Several other levels are observed above 1100 keV in these nuclei.

INTRODUCTION

THE excited states of deformed nuclei near the boundaries of the deformed regions have been the subject of intensive experimental investigation in recent years. This has come about as a natural extension of the success of the Bohr-Mottelson description[1] of highly deformed nuclei, in order to understand more clearly the relationship between the collective states of spherical and deformed nuclei. The stable isotopes of samarium ($Z=62$) are very appropriate for such a study since they extend from the $N=82$ closed neutron shell to $N=92$ which is well into the region of deformation beyond $N=89$. Here we are concerned with the two deformed even isotopes Sm^{152} and Sm^{154}. Further results concerning the odd isotopes of samarium will be forthcoming in a later paper.

While the low-lying levels of Sm^{152} are relatively well studied[2–8] through the decay of Eu^{152} and of Eu^{152m}, only Coulomb excitation data[9–12] have been previously available for states in Sm^{154} above the 2+

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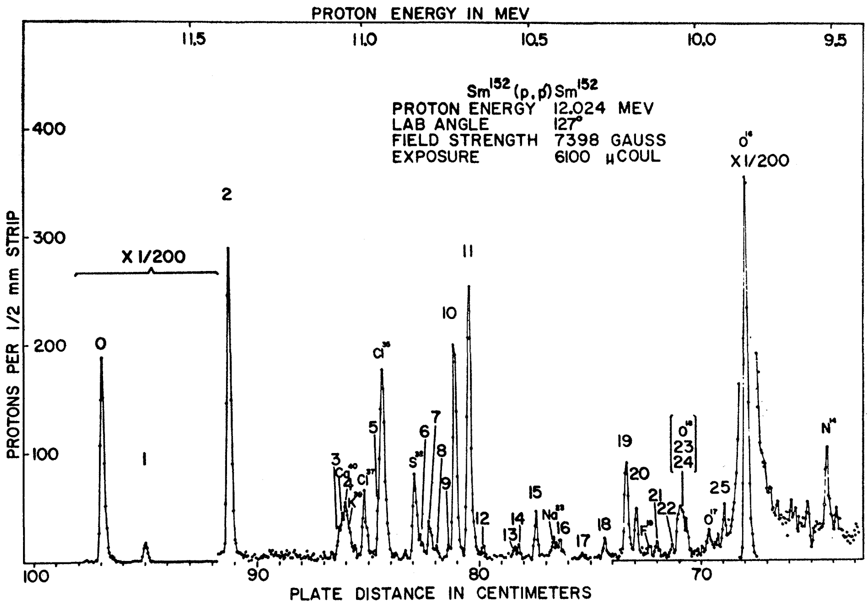

FIG. 1. Representative (p,p') spectrum from the Sm^{152} target. Target groups are numbered sequentially and strong impurity groups are identified.

first excited state. The angular distribution for the latter state has been studied at approximately the same incident proton energy as in the experiments discussed here.[13] We have studied the excited states of these nuclei up to 2-MeV excitation energy through the inelastic scattering of protons at or near the Coulomb barrier (incident proton energy of 12 MeV). Thus we may expect to excite many states which would not be observed in the previous Coulomb excitation or beta-decay studies.

EXPERIMENTAL METHOD

Several detailed descriptions of the experimental method have been previously given.[14–16] Only the most important features will be described here.

A beam of protons from the Florida State University tandem Van de Graaff accelerator is focused on a thin target of enriched samarium oxide which has been evaporated onto a carbon foil of approximately 25-$\mu g/cm^2$ thickness. The enrichments of the samarium oxides used for targets in these experiments are given in Table I. Target thicknesses were not accurately determined for these experiments but ranged from 50 to 200 $\mu g/cm^2$.

The scattered protons were analyzed in a magnetic spectrometer of the Browne-Buechner type[17] and the data were collected on Eastman NTA nuclear emulsions of 50μ thickness. The emulsions were then scanned in 0.5-mm strips on microscopes with calibrated stages.

Over-all resolution in these experiments varied from 0.08 to 0.15% in energy, depending upon the particular conditions. Excitation energies were determined relative to the elastic scattering peaks from the target material and from various impurities present in the targets and in the carbon backings. The spectrograph was calibrated using α sources, elastic scattering and protons from (d,p) reactions for which the Q values are precisely known.

TABLE I. Compositions of the enriched samarium oxides used in target fabrication.

Enriched isotope	Percentage composition of isotopes 144	147	148	149	150	152	154
Sm^{152}	0.03	0.20	0.20	0.27	0.30	97.20	1.80
Sm^{154}	0.02	0.12	0.09	0.12	0.08	0.50	99.07

RESULTS

Inelastic proton spectra were taken at laboratory angles of 90, 95, 115, 127, and 133 deg. A representative proton spectrum from a target enriched in Sm^{152} is shown in Fig. 1. A similar spectrum for Sm^{154} is shown in Fig. 2. The extremely high intensity of the elastic scattering peak from O^{16} and the low intensity of the inelastic protons from samarium precluded any study of excited states above about 2-MeV excitation. Considerable interference with the study of the inelastic groups was caused by elastic scattering peaks from the following nuclei: Ca^{40}, K^{39}, Cl^{37}, Cl^{35}, S^{32}, Si^{28}, O^{18}, and O^{17}. These impurities are found in the target material, the carbon foil and in the detergent employed to float the targets preparatory to mounting. It is possible that

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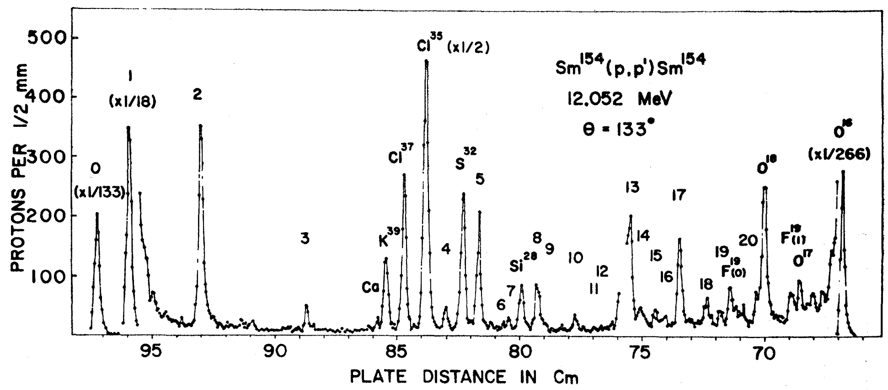


FIG. 2. Representative (p,p') spectrum from the Sm^{154} target. Target groups are numbered sequentially and strong impurity groups are identified.

these elastic scattering peaks may have obscured excited states in the samarium spectra. However, the kinematic shift of these peaks will usually allow the observation of the obscured excitation region at another reaction angle. This is not completely satisfactory because the cross section at the alternate angle could be close to a minimum for the possible excited state in question. It is for this reason that spectra were taken at five different reaction angles. With this reservation concerning the interference from impurities, we can state with reasonable certainty that we have observed the states in Sm^{152} and Sm^{154} that are excited by protons at 12 MeV.

The levels placed at 939 and 988 keV in Sm^{152} require special comment, since these states were not observed in previous experiments and are not easily accounted for by theory. First of all, these weak groups are not in the positions which impurities might occupy. In addition, the group positions agree in excitation, within a few keV, between the various observation angles. These groups do not correspond well in position to the known intense groups from the other Sm isotopes and are, in any case, many times more intense than the percentage composition of the target would allow. Therefore it is concluded that these weak groups represent inelastic scattering from Sm^{152}. The above considerations are indicated in Fig. 3.

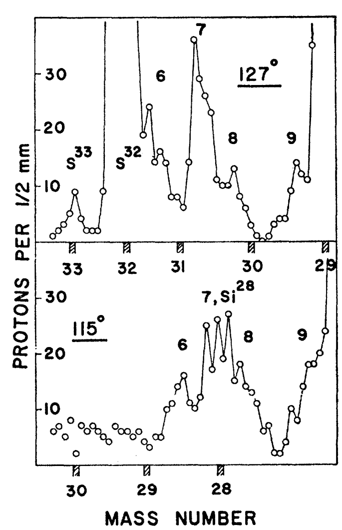


FIG. 3. Expanded view of the region around 960 keV in two Sm^{152} spectra. The expected locations of possible groups from impurity elastic scattering are indicated.

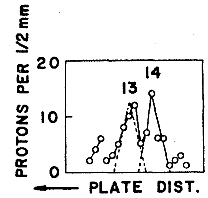
Evidence for doublet structure in the vicinity of 1240-keV excitation in Sm^{152} is shown ln Fig. 4. The observed spectrum is fitted with two peaks of the standard shape which is observed for this region of the focal curve.

An experimental error of ± 10 keV is estimated for the excitation energy of states above 1 MeV. Between 500 keV and 1 MeV the error is estimated at ± 5 keV, and below 500 keV it is estimated at ± 2 keV. This error estimate is considerably greater than the standard deviation of excitation energies observed between spectra taken at different reaction angles and includes an estimate of systematic error. The comparisons necessary to make these error estimates have been made possible by the precise observation of de-excitation gamma rays with bent-crystal and conversion electron spectrometers.

LEVEL SCHEMES

Sm^{152}. A level scheme based on the results discussed above is shown in Fig. 5. These inelastic proton studies have confirmed all of the levels observed in the decay of Eu^{152} and Eu^{152m} except a 2− level at 1537 keV. In addition the 6+ rotational state in the ground-state band, the 4+ rotational state built on the 2+ gamma vibration and the 3− octupole state have been observed. The situation with respect to the 3− state is not completely clear, however (see Discussion below). A large number of additional levels have been observed in

FIG. 4. Expanded view of the 1230-keV excitation region to show the evidence for two close-lying states at 1226 and 1235 keV.



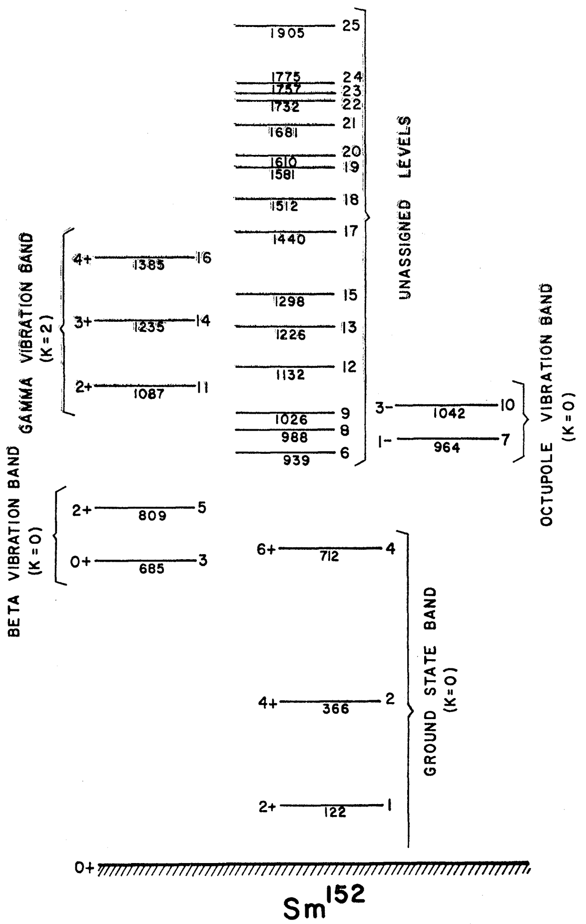


FIG. 5. A level scheme for Sm^{152} based on the present work. The level number to the right of each level corresponds to Fig. 1. The indicated spins and parities have been determined from other experiments. Excitation energies are given in keV.

the region below 2-MeV excitation. A very prominent feature in the inelastic proton spectra is the large cross section of the 1042- and 1087-keV states.

Sm^{154}. Multiple Coulomb excitation results on the excited states of Sm^{154} have recently been published.[9–12] The present studies confirm the existence of these levels and show several additional states below 2 MeV. The resulting level scheme is shown in Fig. 6. The states analogous to the two strongly excited states in the spectrum of Sm^{152} are at 1014 and 1444 keV in the Sm^{154} spectrum. Two other fairly strong states are located at 1181 and 1589 keV. It should be noted that the excitation region between 600 and 900 keV was incompletely studied due to interference from impurity elastic scattering.

DISCUSSION

The Sm isotopes are situated in a unique position in the nuclear periodic table, spanning the region from the $N=82$ closed neutron shell to strongly deformed nuclei (i.e., Sm^{154}). Thus, in going from Sm^{148} and Sm^{150} [15] to Sm^{152} and Sm^{154} in this study, the drastic effect on collective level systematics of going from spherical to spheroidal nuclei has again been demonstrated. This is shown in Fig. 7.

The levels in Sm^{152} are particularly interesting because they represent a transition between highly deformed and spherical nuclei. This transition is much more sudden in the Sm region than in the Os region in that of the even-even nuclei only those with neutron number $N=90$ show an intermediate spectrum. These nuclei, in addition to the Os isotopes, represent a testing ground for nuclear models because deviations from pure rotational bands can be expected to be large. They include Nd^{150}, Sm^{152}, Gd^{154}, and Dy^{156}.

It has been well known[18] that the excited states of

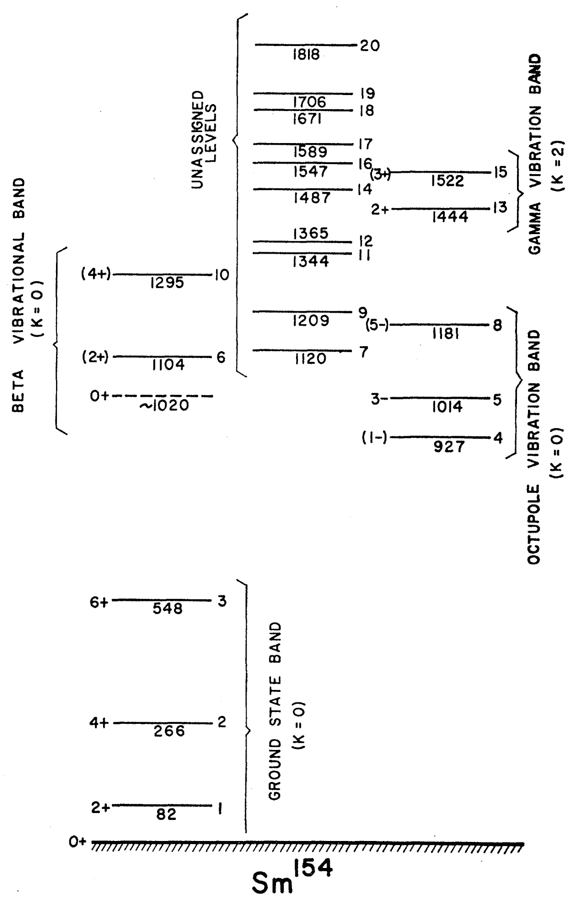


FIG. 6. A level scheme for Sm^{154} based on the present work. The level number to the right of each level corresponds to Fig. 2. Spins and parities from other experiments are shown. Additional spins and parities suggested by collective level systematics are shown in parentheses. The excitation energies are given in keV. The 0+ state at approximately 1020 keV was not observed in these experiments.

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Sm^{152} cannot be exactly fitted by the asymmetric rotor model of Davydov and Fillipov[19] or the extension of this model by Davydov and Chaban.[20] However, it should be noted that the range of variation of the rotor parameters gamma and mu in the latter model required to fit the observed ground-state band and the gamma band is small. This may be taken as an indication of the general validity of the model although, of course, it diminishes the usefulness of the model in identifying excited states on the basis of energy alone.

Recently, it has also become apparent that in the alternative description of the collective states by Bohr and Mottelson,[1] the usual expression for the energy of a rotational state built on the 0+ ground state

$$E=(\hbar^2/2J)I(I+1)-BI^2(I+1)^2$$

is inadequate. An additional term, which has been found to be appropriate for other nuclei,[21] is also necessary in this case. The relationship

$$E=(\hbar^2/2J)I(I+1)-BI^2(I+1)^2+CI^3(I+1)^3$$

fits the ground-state band energies with the values $A=\hbar^2/2J=21.43$, $B=0.102$, and $C=2.27\times10^{-3}$. This predicts the 8+ state at 1343 keV. That these experiments do not show such a state is not too surprising when the relative intensities of the 2+, 4+, and 6+ states of the ground-state band are considered. Also, the cubic term is not well determined from these data.

It seems probable from these data and the data of Hansen and Nathan that there exists a near degeneracy between the 3− octupole state and the 4+ rotational state built on the 0+ beta vibration. It is very difficult to reconcile the small cross section for the 1026-keV state (which is considered to be the 1017-keV state observed by multiple Coulomb excitation[11]) with a 3− assignment. The analogous state in Sm^{148}, Sm^{150}, and Sm^{154} is strongly excited by incident protons at this bombarding energy. Therefore, by these considerations, the 1043-keV state is believed to be the 3− state observed by Hansen and Nathan. The 4+ state in the beta band is believed to be unresolved from the 3− state in this work.

A prediction that $B(E2)_{2+(\gamma)}\approx0.05B(E2)_{2+(\text{g.s.})}$ is made by the asymmetric rotor model for the nucleus Sm^{152}. It is known that in the approximation of completely direct interactions that the inelastic proton scattering cross sections are proportional to $B(E\lambda)$.[22] This allows a rough estimate to be made for the relative intensities of the first and (presumably) second excited 2+ states since they should have similar angular distributions. The resulting ratio $R=0.06\pm0.01$ indicates that the state at 1087 keV is in fact the 2+ gamma vibrational state.

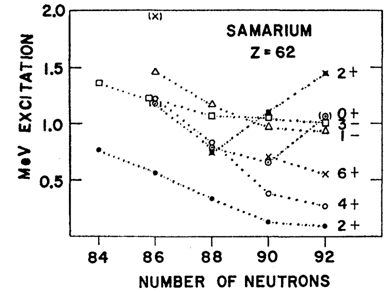

FIG. 7. Systematics of the collective levels in the even-even isotopes of samarium. Uncertain assignments are in parentheses. The transition from spherical to deformed characteristics between Sm^{150} and Sm^{152} is very clear for the positive parity states. Negative parity states vary in a smooth manner between these two nuclei.

The character of the extremely weak states observed near the well-known 963-keV state is not completely clear. One possibility is that they form the states in a rotational band built on a two-quasiparticle intrinsic state with $K=0$. This type of excitation has been previously considered[23] and the possible pairs of nucleons giving rise to such a state are (in the notation of Nilsson[24]) as follows: 642↑−523↓ (neutron pair), 651↑−521↓ (neutron pair) and 413↑−532↓ (proton pair). In this interpretation the levels observed would be 939 (0−), 964(1−), 988(2−) and 1043(3−) keV. In the strict rotational approximation, the ratio of the energy spacing of the (1−) and (3−) levels to the (0−) and (2−) spacing would be 1.666 while our observed ratio is 1.59 ± 0.15. The relative displacement of the odd and even members of such a band is not unexpected[25] and depends on the coupling between the last two protons or neutrons in the particular configuration. Unfortunately we do not observe any other levels with excitation energies appropriate to higher rotational states in such a band. This weakens the argument for a two-quasiparticle interpretation of these states. Furthermore the unusually low value for $\hbar^2/2J$ ($\sim$8 keV) throws some doubt on this interpretation. The occurence of an intrinsic excitation at such a low energy is most surprising and further experiments to check the existence of the 939- and 988-keV states would be most useful.

The 3+ and 4+ members of the gamma vibrational band have been observed at 1235 and 1385 keV, respectively, in these experiments. Applying a rotation-vibration correction to these energies then predicts the 5+ level to be in the vicinity of 1700 keV. Several states are observed in that region of the inelastic spectrum.

In the absence of more detailed information concerning spins and parities it is difficult to discuss the newly observed states above 1 MeV. However, the states at

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1581 and 1681 keV are in the approximate positions for rotational states with spins 2 and 3, respectively, based on the well-known 1512(1−) keV state.

Further experiments to verify the apparent doublet structure at 1226–1235 keV are important.

It is an interesting fact that both the Bohr-Mottelson and Davydov models fail, unless relatively high-order rotation-vibration interaction terms are included, to fit the energies of the ground-state band. In this connection a careful calculation has been done in which rotation-vibration is taken into account to three phonon order.[26] Agreement with the higher rotational states in Sm^{152} is considerably improved, although it is poorer for this transitional nucleus than for any other deformed nucleus.

In Sm^{154}, as in Sm^{152}, the ground-state rotational band is excited up to spin 6; no higher rotational states are definitely observed. The previously discussed cubic expansion with the constants $A=\hbar^2/2J=13.87$, $B=0.037$ and $C=4\cdot10^{-4}$ obtained from these states predicts the spin 8 and 10 states at 960 and 1620 keV, respectively. A state at approximately 960 keV has been observed in heavy ion multiple Coulomb excitation and has been tentatively identified as the 8+ rotational state.[9]

The next step in the interpretation of our data is the identification of the 3− octupole state. On the basis of relative cross sections and the previous measurements of Hansen and Nathan we identify the intense 1014-keV group as the 3− collective vibration. The 1− octupole state will then lie below 1014 keV and is almost certainly the 927-keV level; it is then observed that the 927-, 1014-, and 1181-keV levels fit the energy spacing for a 1−, 3−, 5− rotational sequence. The moment of inertia is consistent with other known octupole bands in the rare-earth region.

In analogy with the (p,p') spectra for Sm^{152} where the gamma band head is strongly excited, a tentative assignment of 2+ can be made for the strong 1444-keV state in Sm^{154}. This is in agreement with another, more definitive, study by multiple Coulomb excitation.[10] Rotational systematics then indicate that the 1522- and 1589-keV levels are the 3+ and 4+ members of the gamma band. The small departure from a strict $I(I+1)$ relationship in this band may be due to perturbations from nearby intrinsic states of the same spin and parity. An analysis of gamma vibrations by Bes[27] shows that the gamma vibrational state should be located at approximately 1500 keV in Sm^{154}. Thus the observed γ vibrational energy is in relatively good agreement but somewhat lower than the calculated energy, as has been noted previously. If we identify the 1444-keV state as the gamma vibration and assume that the inelastic cross sections of this state and the first excited 2+ state are proportional to $B(E2)$, then the ratio $R=B(E2)_{2+(\gamma)}/B(E2)_{2+(\mathrm{g.s.})}=0.03\pm0.01$ as compared with $R=0.06\pm0.01$ in the case of the analogous states in Sm^{152}. The two states are assumed to have similar angular distributions in this calculation. The values for R predicted by the Davydov and Fillipov model are 0.05 and 0.03, in good agreement with the data even though strict Coulomb excitation conditions are not fulfilled.

No definite evidence for assignment of the beta vibrational band head was obtained in these experiments. However, other work has shown[10] that the band head lies at about 1020 keV. Since these experiments excited the 0+ beta band head in Sm^{152}, the corresponding state in Sm^{154} must lie within 10 keV of the strong 1014-keV state. This implies that the 2+ beta band state must be either the 1104- or 1120-keV level. The former is somewhat more likely to be this state since then rotational systematics would place the 4+ state at the observed energy of 1295 keV for the only possible candidate for this level.

Further interpretation of the levels of Sm^{154} will require more detailed experiments which determine directly the spins and parities of the excited states and which test in some manner whether they are collective or particle in nature. The same two-quasiparticle states as those listed previously for Sm^{152} are available for Sm^{154}.

ACKNOWLEDGMENTS

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